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Short communication

A large-strain finite element formulation for biological tissues with application to mitral valve leaflet tissue mechanics

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Abstract

This paper presents a finite element formulation suitable for large-strain modeling of biological tissues and uses this formulation to implement an accurate finite element model for mitral valve leaflet tissue. First, an experimentally derived strain energy function is obtained from literature. This function is implemented in finite elements using the mixed pressure-displacement formulation. A modification is made to aid in maintaining positive definiteness of the stiffness matrix at low strains. The numerical implementation is shown to be accurate in representing the analytical model of material behavior. The mixed formulation is useful for modeling of soft biological tissues in general, and the model presented here is applicable to finite element simulation of mitral valve mechanics.

Keywords: Biological tissues; Mitral valve; Large strain; Constitutive models; Finite element formulation

1. Introduction

Much research has been done in determining material constitutive models for soft biological tissues. We are interested in modeling heart mitral valve leaflet tissue and there are a number of relevant works. A comprehensive review of the models for soft biological tissue in general is available in the literature (Sacks and Sun, 2003), so is the review specific to heart valve tissue (Weinberg and Kaazempur-Mofrad, 2005), and reviews for similar tissues such as the blood vessel wall (Vito and Dixon, 2003). Most of these models agree on the basic assumptions of material behavior. Researchers have shown that the aligned fibrous structure of these tissues gives rise to anisotropic hyperelasticity in the physiological ranges of strain rate (Carew et al., 1968). Additionally, the observation that a significant portion of the tissue volume is composed of water that appears to be tightly bound to the solid matrix leads to the assumption that such tissues are incompressible (Weiss et al., 1996; Holzapfel and Eberlein et al., 1996; May-Newman and Yin, 1998).

Two main types of approaches to modeling these tissues have been rigorously applied in literature, one structurally based and one invariant based. The structurally based model allows for splaying of fibers, and incorporates the effects of all the fibers in different directions by integral (Sacks, 2003). This method has been shown to be accurate in aortic valve tissue (Billiar and Sacks, 2000). Alternatively, the model can be based on invariants and the assumption of material transverse isotropy. May-Newman and Yin (1995, 1998) have experimentally derived such a model for mitral valve leaflet tissue behavior and shown it to be accurate. In this paper, we use the invariant-based approach since a proven constitutive model is available for the tissue in which we are interested and since invariant-based approach requires fewer calculations than the

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structurally-based, and thus will be more computationally efficient. An invariant-based approach describes transverse isotropy by use of directional pseudo-invariants (Spencer, 1972).

We implement the model in the finite element setting by following the mixed pressure–displacement (u/p) formulation of Sussman and Bathe (1987). A number of similar finite element implementations have been published for incompressible, transversely isotropic materials (Holzapfel, 2001; Almeida and Spilker, 1998; Rüter and Stein, 2000; Schröder and Neff, 2003). We introduce a modification to the strain energy function in order to maintain positive definiteness of the stiffness matrix at low strains. The implemented model is verified by comparing the numerical solution to analytical results, showing that the implementation accurately represents the original strain energy function.

2. Continuum mechanics definitions

All calculations here are performed in terms common to large-strain continuum mechanics. The deformation gradient is denoted as

$$\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}, \tag{1}$$

where $\boldsymbol{X}$ is the original (undeformed) configuration and $\boldsymbol{x}$ is the deformed configuration. The right Cauchy–-Green deformation tensor is

$$\boldsymbol{C} = \boldsymbol{F}^{\mathrm{T}} \cdot \boldsymbol{F}, \tag{2}$$

the strain invariants in terms of $\boldsymbol{C}$ are given by

$$\begin{aligned} I_1 &= \operatorname{tr} \boldsymbol{C}, \\ I_2 &= \frac{1}{2}((\operatorname{tr} \boldsymbol{C})^2 - \operatorname{tr} \boldsymbol{C}^2), \\ I_3 &= \det \boldsymbol{C} \end{aligned} \tag{3}$$

and the Jacobian is $J = \sqrt{I_3}$. Transverse isotropy is incorporated into the model by introducing a vector that defines the preferred fiber direction of the material. Denoting the vector as $\boldsymbol{N}$, the stretch in the fiber direction is

$$\alpha = \sqrt{\boldsymbol{N} \cdot \boldsymbol{C} \cdot \boldsymbol{N}}, \tag{4}$$

and two pseudo-invariants can be defined in terms of the right Cauchy–Green strain (Spencer, 1972):

$$\begin{aligned} I_4 &= \boldsymbol{N} \cdot \boldsymbol{C} \cdot \boldsymbol{N} = \alpha^2, \\ I_5 &= \boldsymbol{N} \cdot \boldsymbol{C}^2 \cdot \boldsymbol{N}. \end{aligned} \tag{5}$$

The stress state is calculated from the deformation state based on a strain energy function, W:

$$\boldsymbol{S} = 2\frac{\partial W}{\partial \boldsymbol{C}}, \tag{6}$$

where $\boldsymbol{S}$ is the 2nd Piola–Kirchoff stress tensor, and the Cauchy stresses can be determined by

$$\boldsymbol{\sigma} = J^{-1}\boldsymbol{F} \cdot \boldsymbol{S} \cdot \boldsymbol{F}^{\mathrm{T}}. \tag{7}$$

The material constitutive tensor is

$$\boldsymbol{C} = 4\frac{\partial^2 W}{\partial \boldsymbol{C}^2}. \tag{8}$$

The three invariants described in Eq. (3) are recognized to describe isotropic hyperelasticity. Spencer (1972) has shown that the full set of five invariants, defined in Eq. (3) and (5) can be used to describe the strain energy function of transversely isotropic hyperelasticity.

3. Experimentally determined strain energy function for mitral valve leaflet tissue

A strain energy function for mitral valve leaflet tissue was carefully determined and verified by May–Newman and Yin (1995, 1998). The mitral tissue's stress-deformation response was shown to be chiefly a function of the first invariant and the stretch in the fiber direction,

$$W = W(I_1, \alpha). \tag{9}$$

Specifically, the response was modeled by a form analogous to the exponential proposed by Fung (1967),

$$W(I_1, I_4) = c_0\left\{\exp[c_1(I_1 - 3)^2 + c_2(I_4^{1/2} - 1)^4] - 1\right\}, \tag{10}$$

where c_0, c_1, and c_2 are constants fit to the experimental data, and we have used Eq. (5) to substitute I_4 for α. The anterior and posterior leaflets have slightly different responses, reflected by the difference in values for the three constants shown in Table 1.

The strain energy function in Eq. (10) along with the coefficient values in Table 1 accurately predict the stress-deformation behavior of the leaflet tissue.

4. Mixed (u/p) formulation

In the modeling of incompressible and nearly incompressible solid media, a displacement-based finite element formulation gives large errors in the predicted stresses. An involved discussion of these errors is

Table 1
Coefficient values for mitral valve tissue

	c_0 (kPa)	c_1	c_2
Anterior	0.399	4.325	1446.5
Posterior	0.414	4.848	305.4

provided by Bathe (1996). A formulation where the material is considered to be nearly incompressible and the nodal displacements and pressures are separately interpolated is considered to be the most attractive for modeling these materials. Sussman and Bathe (1987) refer to such an approach as a mixed displacement–-pressure (u/p) formulation and provide a general framework. Here, we apply the mixed formulation to the above energy function.

The deformation gradient is determined by standard method from the displacements (see Bathe, 1996 for details), from which the invariants of Eqs. (3) and (5) are calculated. The original strain energy function must be made insensitive to volume changes. This is done by substituting modified invariants for the original invariants (Weiss et al., 1996),

$$J_1 = I_1 I_3^{-1/3},$$
$$J_4 = I_4 I_3^{-1/3} \tag{11}$$

and we follow Sussman and Bathe (1987) in defining $J_3 = \sqrt{I_3}$.

Defined as such, J_1 and J_4 are dependent on deviatoric deformations and independent of volumetric changes. Substituting these modified invariants into the original strain energy function gives a strain energy term that is completely deviatoric and determined entirely by the displacements,

$$\bar{W}(J_1, J_4) = c_0\left\{\exp\left[c_1(J_1-3)^2 + c_2(J_4^{1/2}-1)^4\right] - 1\right\}, \tag{12}$$

where the overbar denotes quantities determined as functions solely of the nodal displacements.

An additional potential is supplied by Sussman and Bathe to include the effect of the interpolated pressure,

$$Q = -\frac{1}{2}(\bar{p} - \tilde{p})^2, \tag{13}$$

where $\tilde{p}$ is the interpolated pressure and $\bar{p}$ is the pressure determined by displacements,

$$\bar{p} = -\kappa(J_3 - 1). \tag{14}$$

The complete strain energy function is then

$$W = \bar{W} + Q. \tag{15}$$

The stresses and material stiffness tensor are found by substituting Eq. (15) into Eqs. (6)–(8), and the steps for assembling the element matrix entries from the strain energy function are given by Bathe (1996).

5. Modification to strain energy function

Efficient solution of the global finite-element matrices may require a positive definite stiffness matrix, so we would like the stiffness matrix derived here to be positive definite. The stiffness associated with the exponential strain energy function approaches the zero matrix at zero strains. We add a term to Eq. (15) to ensure that the matrix can be decomposed at low strains,

$$W^{(\mathrm{PD})}(J_1) = c_{\mathrm{PD}}(J_1 - 3), \tag{16}$$

where c_{PD} is a constant small enough to guarantee that $W^{(\mathrm{PD})} \ll W$, so that $W^{(\mathrm{PD})}$ does not contribute appreciably to the stress response. $W^{(\mathrm{PD})}$ may be recognized as the first term in a standard Mooney–Rivlin model. The strain energy function is now

$$W = \bar{W} + Q + W^{(\mathrm{PD})}. \tag{17}$$

6. Software implementation and verification

The formulation derived above was implemented as a user-supplied material model in the finite element package ADINA (ADINA R & D, Inc. Watertown, MA 02472, USA, 2004). To verify the constitutive model, a unit-length cube of tissue was and meshed with a single 27 node solid element. 27 node interpolation was used for displacements and 8 node interpolation for pressures. Full Gauss integration was used for all terms. The fiber direction N was aligned with the x-axis. Uniaxial and biaxial strain conditions were simulated by applying displacements to the tissue boundaries in the x- and y-directions. Material constants listed in Table 1 were used, along with $c_{\mathrm{PD}} = 10^{-8}$ and $\kappa = 10^6$ Pa.

To verify the material model, the tissue was subjected to biaxial strain conditions replicating experiments run by May-Newman and Yin (1998). The range of stresses in these experiments is consistent with the range of stresses expected in a functioning mitral valve (Kunzelman et al., 1993). Resulting stresses output by the finite element solution were compared to the analytical solution (see Figs. 1–4). The analytical responses were found by applying biaxial strain conditions, solving for the out-of-plane strain directly from incompressibility

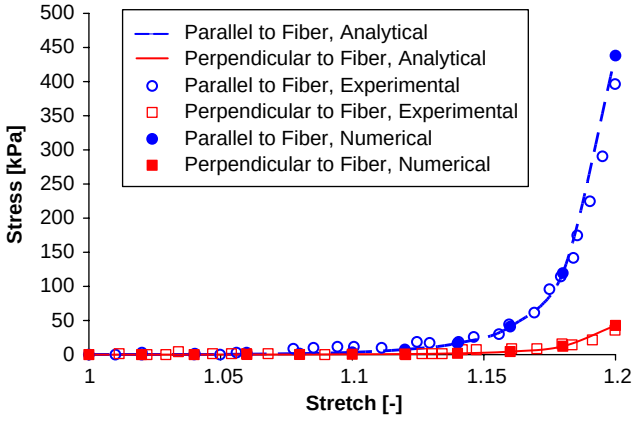

Fig. 1. Equibiaxial strain applied to anterior leaflet.

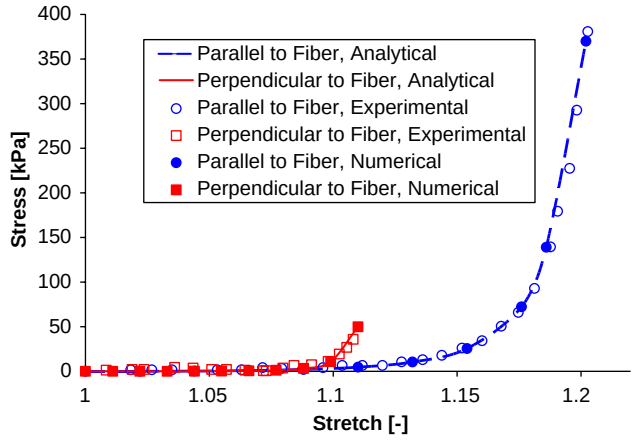

Fig. 2. 2:1 Off-biaxial strain applied to anterior leaflet.

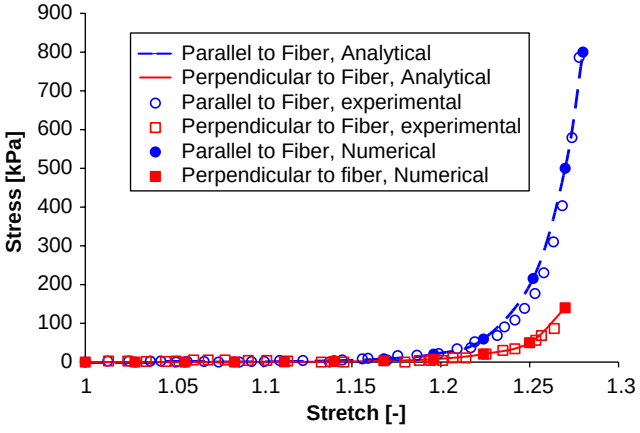

Fig. 3. Equibiaxial strain applied to posterior leaflet.

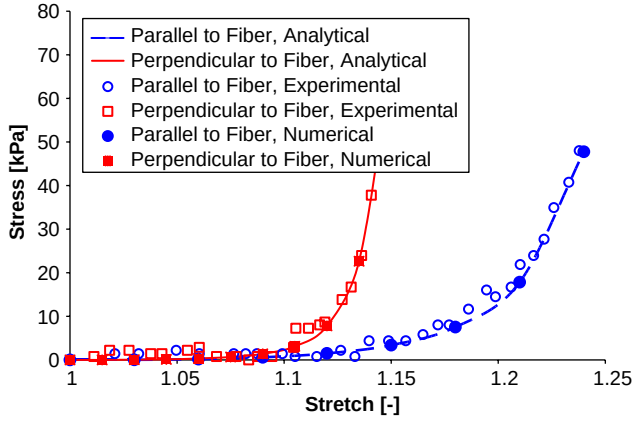

Fig. 4. 2:1 Off-biaxial strain applied to posterior leaflet.

and calculating the stresses from the original strain energy function. We also include the experimental data on the plots. In all cases, the stress in the direction parallel to the fiber is plotted against the stretch in the fiber direction and the stress in the direction perpendicular to the fiber is plotted vs. the stretch perpendicular to the fiber. In these four cases, the result of the finite element solution matches the analytical solution with errors within the limits of numerical accuracy. The stiffness matrix was positive definite at all steps. Within the stress range of these experiments and at stresses up to 5×10^4 kPa, the volume ratio stayed within 0.001% of 1, showing that incompressibility was maintained.

7. Discussion

Data plotted in Figs. 1–4 show that the numerical implementation accurately represents the analytical solution in biaxial conditions. Adding a term to the strain energy to maintain positive definiteness was effective without introducing significant error to the stresses.

The mixed formulation relies on the volumetric stresses being much larger than the deviatoric in order to enforce incompressibility. Due to the high exponential material behavior, strains just beyond those in the protocols used here can cause deviatoric stresses large enough to affect the incompressibility. Our approach still enforces incompressibility at stresses well beyond those expected in a functioning mitral valve. The model presented here is accurate in predicting mitral valve leaflet behavior over the necessary stress range, and thus is useful in further finite-element simulation of mitral valve function.

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